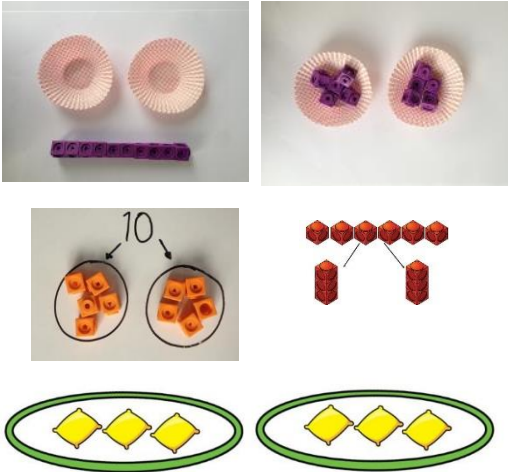
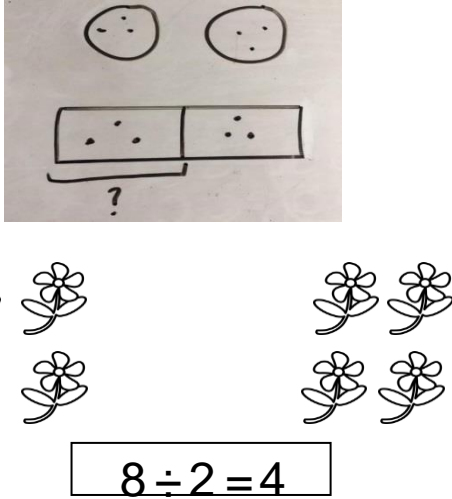

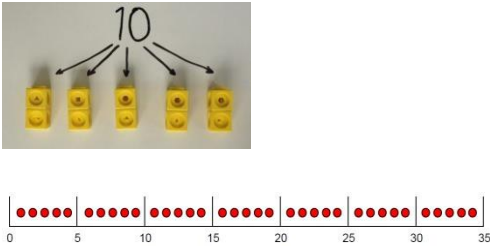
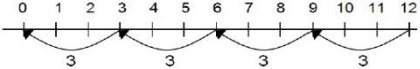
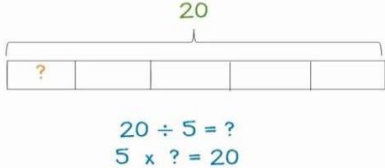
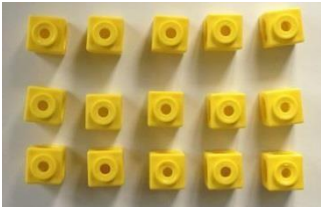
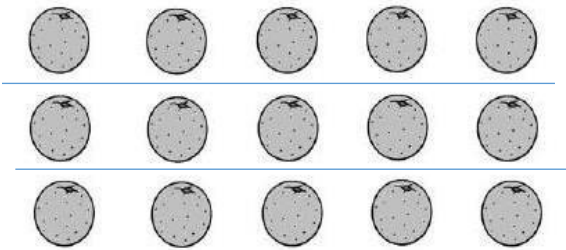
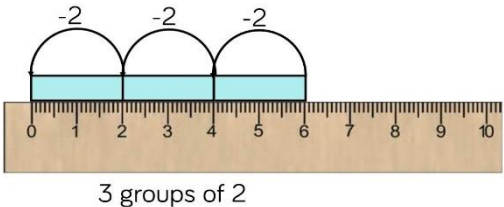
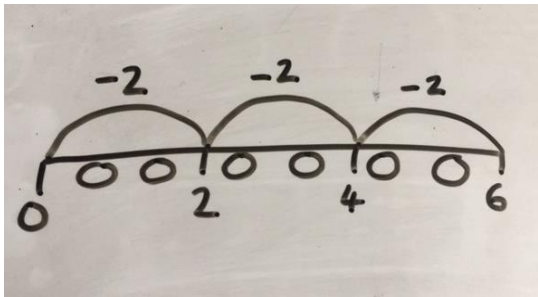
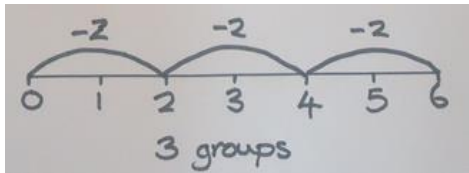


	Objective	Concrete	Pictorial	Abstract
Foundation	<p>Sharing objects into equal groups</p> <p>Focus on terminology of equal groups.</p>	<p>Use a range of objects/resources to share e.g.</p> 	<p>Represent the sharing pictorially.</p>  <p><math>8 \div 2 = 4</math></p>	<p><math>6 \div 2 = 3</math></p>  <p>6 shared into <u>two equal groups</u> is 3.</p>
Year 1	<p>Division as grouping</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p> 	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p><math>20 \div 5 = ?</math> <math>5 \times ? = 20</math></p>	<p><math>20 \div 5 = 4</math> <math>5 \times 4 = 20</math></p>

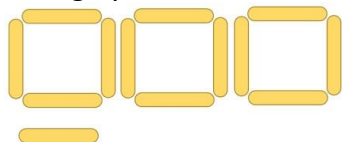
Year 2	Division within arrays – linking to multiplication	<p>Children link division to multiplication by creating an array and thinking about the number sentences that can be created.</p>  <p>E.g. <math>15 \div 3 = 5</math>    <math>5 \times 3 = 15</math>  <math>15 \div 5 = 3</math>    <math>3 \times 5 = 15</math></p>	<p>Children draw an array and use lines to split the array into groups to make multiplication and division sentences.</p> 	<p>Find the inverse of multiplication and division sentences by creating four linking number sentences.</p> <p><math>7 \times 4 = 28</math>  <math>4 \times 7 = 28</math>  <math>28 \div 7 = 4</math>  <math>28 \div 4 = 7</math></p>
	Repeated subtraction	<p>Children use repeated subtraction using base 10 above a ruler/number lines.  <math>6 \div 2 =</math></p> 	<p>Children represent repeated subtraction pictorially.</p> 	<p>Abstract number line to represent the equal groups that have been subtracted.</p> 

Division with remainders.

$2d \div 1d$  with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

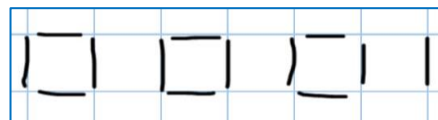
$$13 \div 4$$

Use of lollipop sticks to form wholes-squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

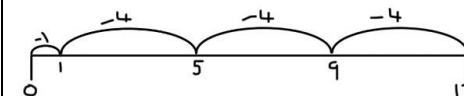


There are 3 whole squares, with 1 left over.

$$13 \div 4 = 3 \text{ remainder } 1$$

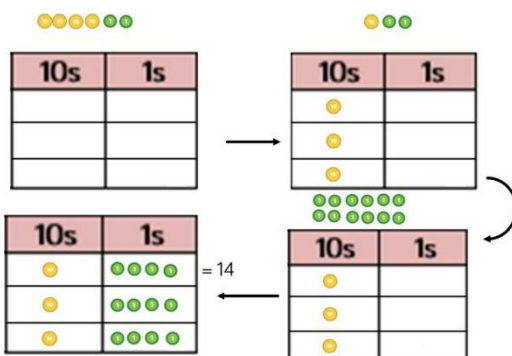
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

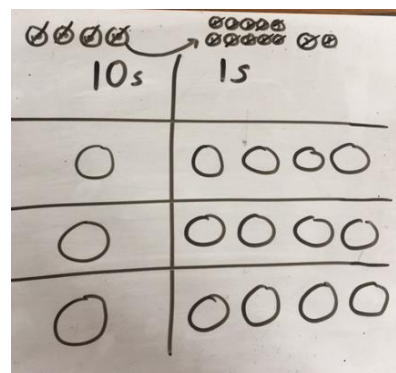


Division by sharing into equal groups to support short division

Sharing using place value counters.  
 $42 \div 3 = 14$



Children pictorially represent using place value counters.



Children to be able to make sense of the place value counters and write calculations to show the process.

$$\begin{aligned} 42 \div 3 \\ 42 &= 30 + 12 \\ 30 \div 3 &= 10 \\ 12 \div 3 &= 4 \\ 10 + 4 &= 14 \end{aligned}$$

Partitioning to support division of dividing 2 and 3 digit numbers by 1 digit numbers (no remainders)

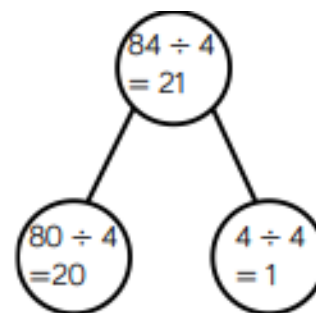
Use of place value counters and base 10 to support partitioning and sharing into equal groups.

$$84 \div 4$$

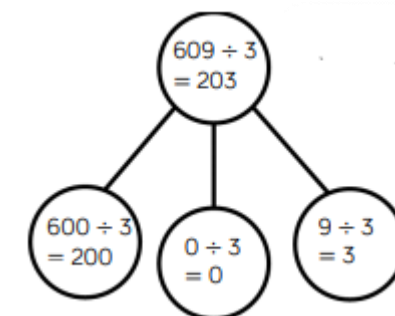
Tens	Ones
10	1
10	1
10	1
10	1

$$609 \div 3$$

Hundreds	Tens	Ones
100		1
100		1
100		1
100		1



$$\begin{aligned} 84 \div 4 &= 21 \\ 80 \div 4 &= 20 \\ 4 \div 4 &= 1 \\ &\rightarrow 20 + 1 = 21 \end{aligned}$$



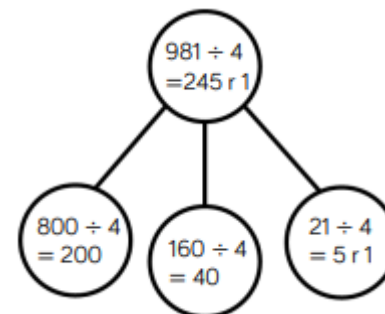
$$\begin{aligned} 609 \div 3 &= 203 \\ 600 \div 3 &= 200 \\ 0 \div 3 &= 0 \\ 9 \div 3 &= 3 \\ &\rightarrow 200 + 0 + 3 = 203 \end{aligned}$$

Partitioning to support division of dividing 2 and 3 digit numbers by 1 digit numbers (with remainders)

Use of place value counters and base 10 to support partitioning and sharing into equal groups.

$$981 \div 4$$

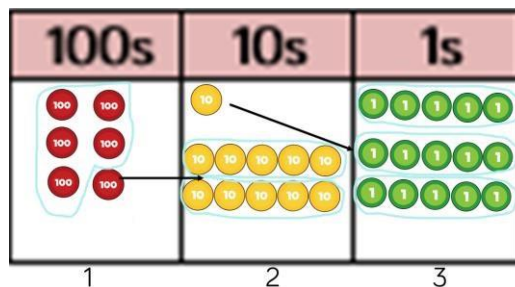
Hundreds	Tens	Ones
100	10	1
100	10	1
100	10	1
100	10	1



$$\begin{aligned} 981 \div 4 &= 245 \text{ r } 1 \\ 800 \div 4 &= 200 \\ 160 \div 4 &= 40 \\ 21 \div 4 &= 5 \text{ r } 1 \\ &\rightarrow 200 + 40 + 5 \text{ r } 1 = 245 \text{ r } 1 \end{aligned}$$

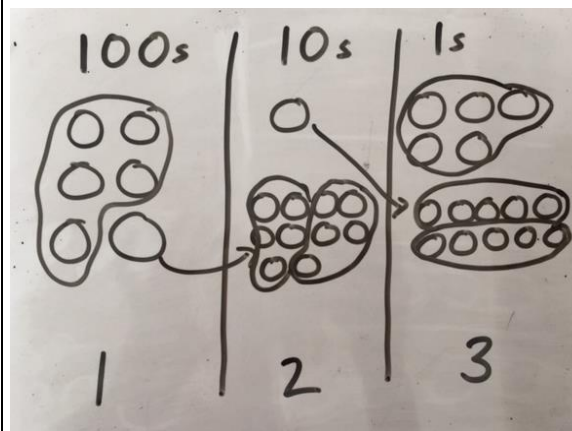
Short division – division of up to 4 digit numbers by 1 digit numbers (no remainders).

Children use place value counters to group.  
 $615 \div 5 =$

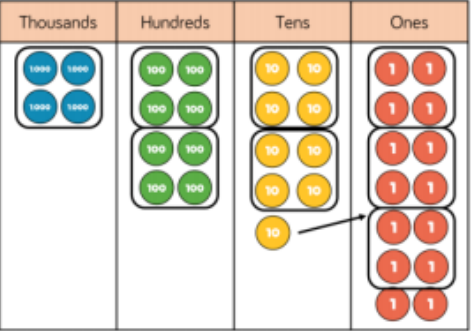
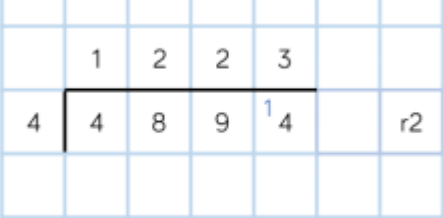

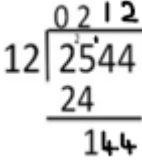


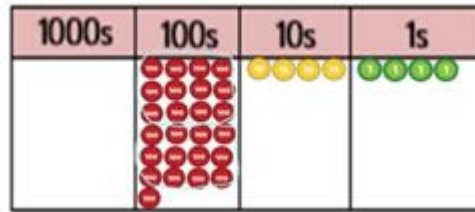
1. Make 615 with place value counters.
2. How many equal groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many equal groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many equal groups of 5 ones can you make with 15 ones?

Children represent the place value counters pictorially.



$$\begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \phantom{00} \\ 11 \phantom{0} \\ \underline{10} \phantom{0} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

	<p>Short division – division of up to 4 digit numbers by 1 digit numbers.</p> <p>Calculations to include remainders.</p>	<p><math>4894 \div 4 = 1223 \text{ r } 2</math></p>  <p>Children to practically complete the exchanges with counters.</p>	<p>Children to draw counters, as in previous example for short division.</p> <p>Exchanges to be completed pictorially.</p>																															
Year 6	<p>Long division – division of up to 4 digit numbers by 2 digit numbers.</p> <p>Calculations with and without remainders.</p>	<p>Long division using place value counters</p> <p><math>2544 \div 12</math></p>  <p>We cannot group 2 thousands into groups of 12, so we exchange them (see next step below).</p>	<p>Children to draw counters, as in examples for short division.</p> <p>Exchanges to be completed pictorially.</p>	 <p>Listing multiples as a supportive strategy.</p> <table data-bbox="1671 1114 1839 1310"> <tr><td></td><td></td><td>0</td><td>3</td><td>6</td></tr> <tr><td>1</td><td>2</td><td>4</td><td>3</td><td>2</td></tr> <tr><td></td><td>–</td><td>3</td><td>6</td><td>0</td></tr> <tr><td></td><td></td><td></td><td>7</td><td>2</td></tr> <tr><td></td><td>–</td><td></td><td>7</td><td>2</td></tr> <tr><td></td><td></td><td></td><td></td><td>0</td></tr> </table> <p>Multiples of 12:</p> <ul style="list-style-type: none"> <li><math>12 \times 1 = 12</math></li> <li><math>12 \times 2 = 24</math></li> <li><math>12 \times 3 = 36</math></li> <li><math>12 \times 4 = 48</math></li> <li><math>12 \times 5 = 60</math></li> <li><math>12 \times 6 = 72</math></li> <li><math>12 \times 7 = 84</math></li> <li><math>12 \times 8 = 96</math></li> <li><math>12 \times 9 = 108</math></li> <li><math>12 \times 10 = 120</math></li> </ul>			0	3	6	1	2	4	3	2		–	3	6	0				7	2		–		7	2					0
		0	3	6																														
1	2	4	3	2																														
	–	3	6	0																														
			7	2																														
	–		7	2																														
				0																														



We can group 24 hundreds into groups of 12, which leaves us with 1 hundred.

Calculations with remainders:  
Pupils to interpret remainders as fractions:  $\frac{9}{13}$ .

			1	0	9	r	9	
1	3	1	4	2	6			
	-	1	3	0	0			(x 100)
			1	2	6			
	-		1	1	7			(x9)
					9			

If the remainder is  $\frac{3}{4}$ , children are to relate this to their knowledge of decimals. E.g. the equivalent remainder is 0.75.

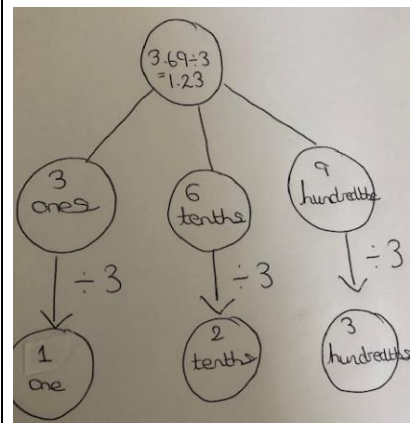
Division of decimal numbers by integers.

Use of base 10 and place value counters to support grouping.  
 $3.69 \div 3 = 1.23$



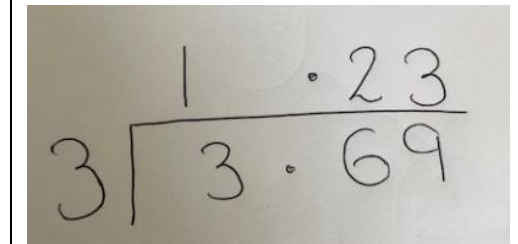
Children to circle groups to support understanding.

Use of the part-whole model to support partitioning.

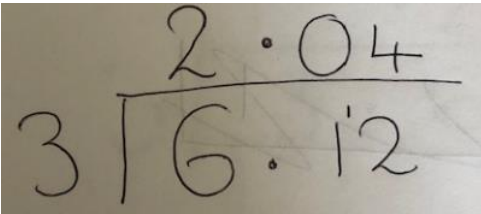


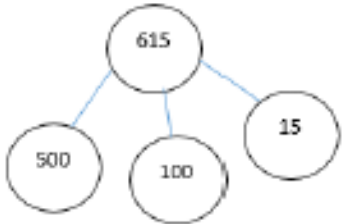

Emphasis on place value and correct vocabulary – e.g. 2 tenths not 2.

$$3.69 \div 3 = 1.23$$





				<p>Example with exchanging in calculation:</p> $6.12 \div 3 = 2.04$ 
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Conceptual variation; different ways to ask children to solve $615 \div 5$			
<p>Using the part whole model below, how can you divide 615 by 5 without using short division?</p> 	<p>I have £615 and share it equally between 5 bank accounts. How much will be in each account?</p> <p>615 pupils need to be put into 5 groups. How many will be in each group?</p>	$5 \overline{)615}$ <p><math>615 \div 5 =</math></p> <p><math>\square = 615 \div 5</math></p>	<p>What is the calculation? What is the answer?</p> 

Key Vocabulary: divide, division, divide by, share, sharing, equal group, half.