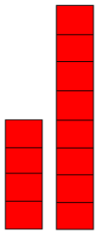





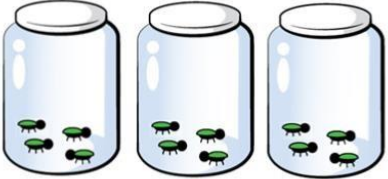

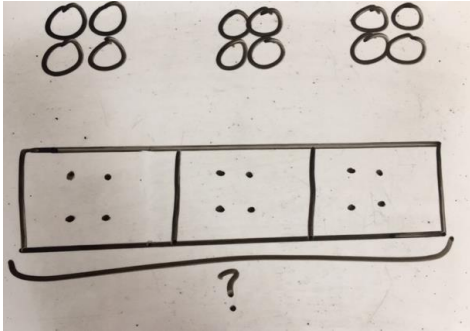
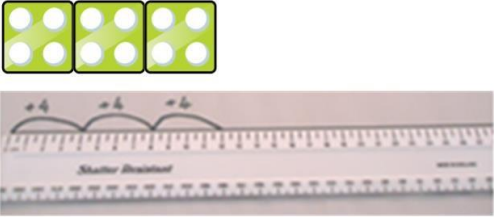
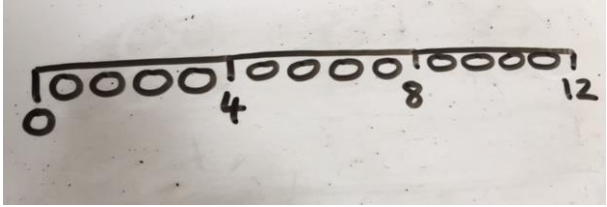
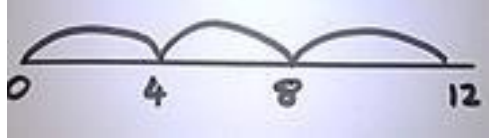


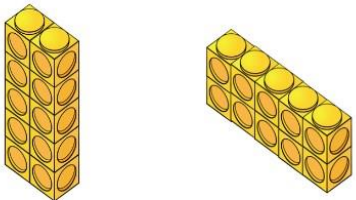
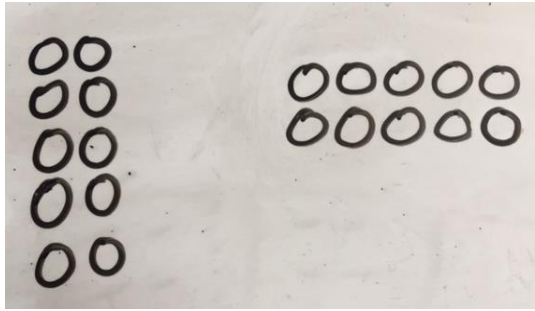
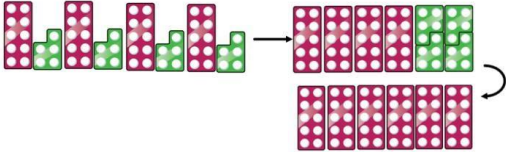
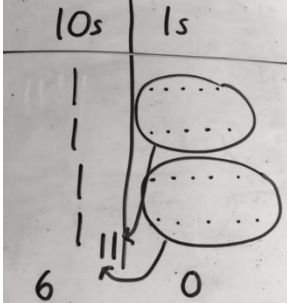
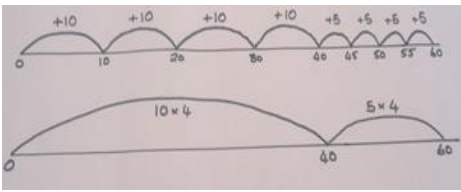


Progression in Calculation: Multiplication



	Objective	Concrete	Pictorial	Abstract
Foundation	Doubling	<p>Use practical resources and activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Children use pictures and jottings to double.</p> <p>Double 4 is 8</p> 	<p>Double 4 is 8</p> <p>$4 + 4 = 8$</p>
Year 1	Counting in multiples	<p>Count in multiples supported by concrete objects in equal groups.</p>   <p>Children use a variety of objects and resources to group.</p>	  <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p> <p>10, 20, 30, 40, 50, 60, 70, 80</p>

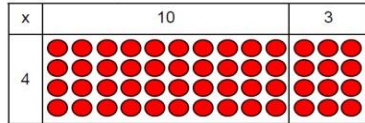
	<p>Repeated addition/ grouping</p>	<p>Children use their knowledge of counting in multiples to understand repeated grouping/addition.</p>  <p>There are 3 equal groups, with 4 in each group.</p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p>$3 \times 4 = 12$</p> <p>$4 + 4 + 4 = 12$</p>
<p>Year 2</p>	<p>Repeated addition with a number line</p>	<p>Children demonstrate repeated addition on a number line with practical resources.</p> <p>$4 \times 3 =$</p> 	<p>Children represent this pictorially alongside a number line.</p> <p>$4 \times 3 =$</p> 	<p>Abstract number line showing three jumps of four.</p> <p>$4 \times 3 = 12$</p> 

	<p>Arrays</p>	<p>Use arrays to illustrate commutativity. Counters and other objects can also be used. $2 \times 5 = 5 \times 2$</p>  <p>2 lots of 5 5 lots of 2</p>	<p>Children represent this pictorially.</p> 	<p>Children to be able to use an array to write a range of calculations e.g.</p> <p> $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$ </p>
	<p>Partition to multiply</p>	<p>Partition to multiply using numicon, base 10 etc. $4 \times 15 =$</p> 	<p>Children to represent the concrete manipulatives pictorially.</p> 	<p>Children to be encouraged to show the steps they have taken.</p> <p> 4×15 $\swarrow \searrow$ $10 \quad 5$ </p> <p> $10 \times 4 = 40$ $5 \times 4 = 20$ $40 + 20 = 60$ </p> <p>A number line can also be used.</p> 

Grid method (area model)

(2 and 3 digit x 1 digit)

Show the link with arrays to first introduce the grid method. Ensure grid method reflects the correct proportions.



4 rows of 10, 4 rows of 3

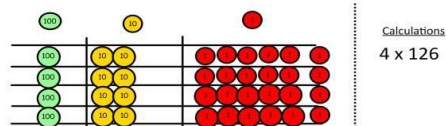
Move on to using Base 10 to move towards a more compact method.



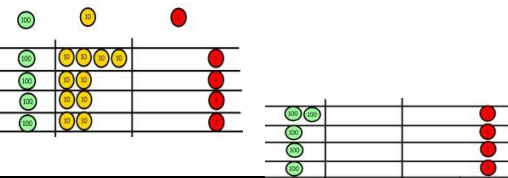
4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.

Fill each row with 126.

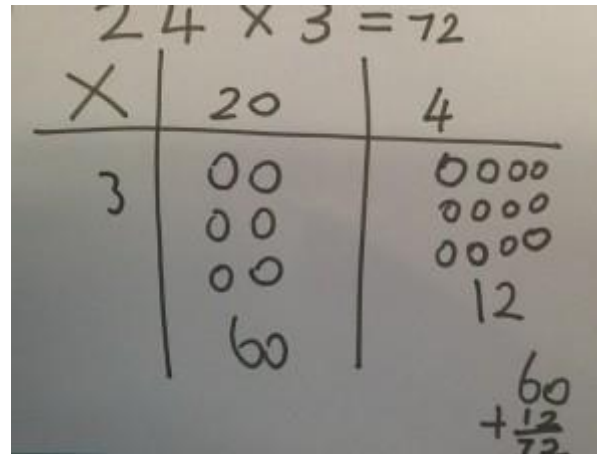


Add up each column, starting with the ones making any exchanges needed.



Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	30	5
7	210	35

$$210 + 35 = 245$$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method (area model).

	10	8
10	100	80
3	30	24

x	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

Column method with expanded layout.

(Up to 4 digit numbers multiplied by 1 or 2 digits)

Use place value counters/base 10.

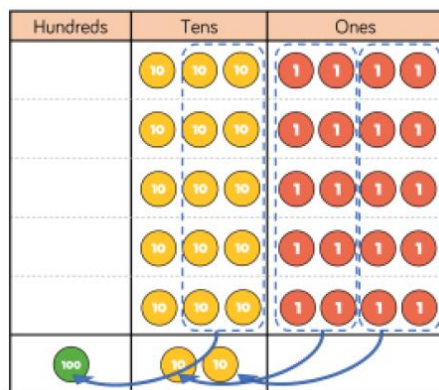
$$3 \times 23$$



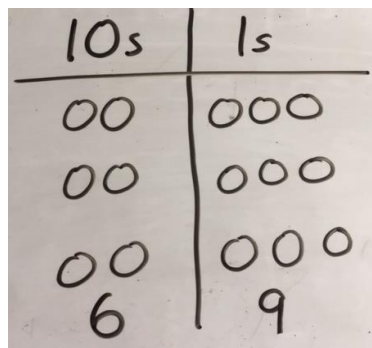
6 9

Use of place value counters/ base 10 to solve 34×5 .

Calculations to involve exchanges.



Children represent the counters pictorially.



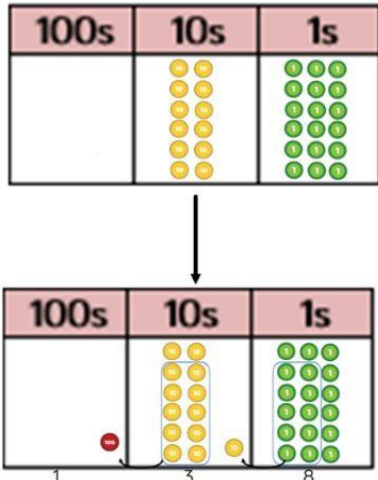
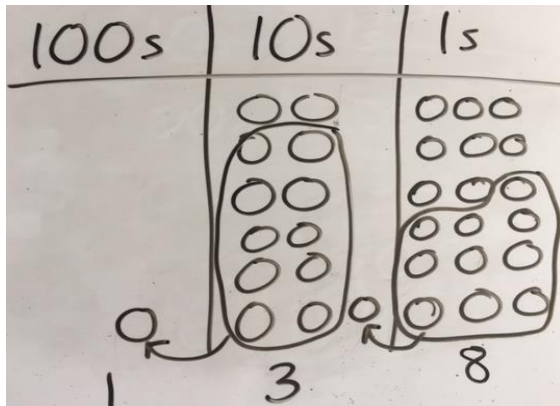
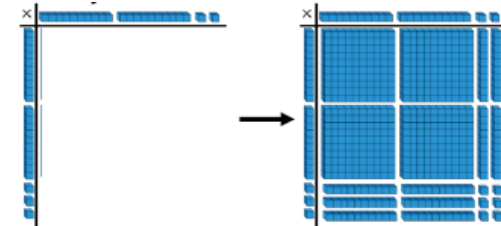
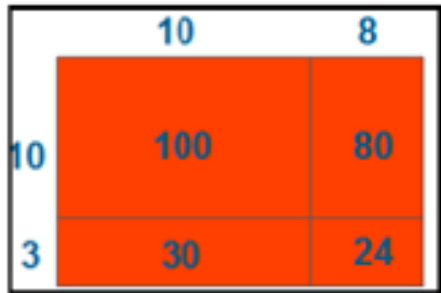
Children record what it is they are doing to show understanding.

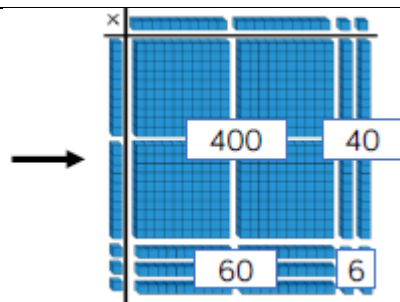
$$\begin{array}{r} 3 \times 23 \\ 20 \quad 3 \end{array}$$

$$\begin{array}{l} 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$$

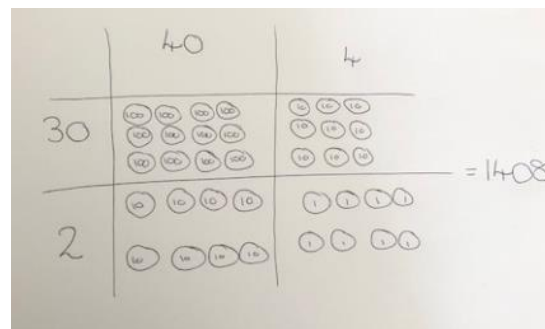
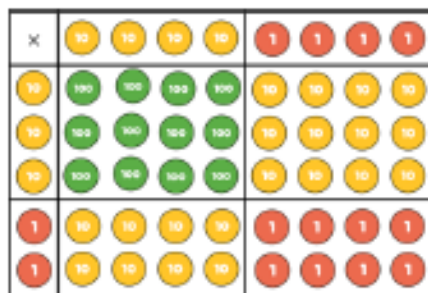
Expanded form for multiplication:

	H	T	O		
		3	4		
x			5		
		2	0	(5 x 4)	
+	1	5	0	(5 x 30)	
	1	7	0		

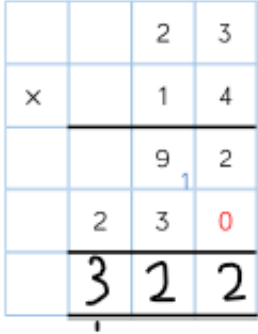
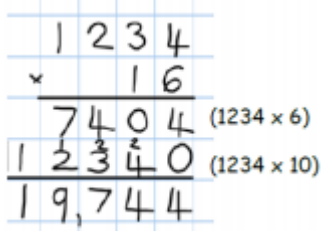
	<p>Short multiplication</p> <p>(Up to 4 digit numbers multiplied by 1 digit number).</p>	<p>Use place value counters/base 10 to model exchanges.</p> <p>6 x 23</p> 	<p>Children represent the counters/base 10 pictorially.</p> 	<p>Formal written method</p> $6 \times 23 =$ $\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ 11 \end{array}$ <table border="1" data-bbox="1682 598 1888 829"><tr><td></td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td>2</td><td>3</td><td>4</td></tr><tr><td>x</td><td></td><td></td><td>6</td></tr><tr><td></td><td>1</td><td>4</td><td>0</td></tr><tr><td></td><td>2</td><td>2</td><td></td></tr></table>		H	T	O		2	3	4	x			6		1	4	0		2	2	
	H	T	O																					
	2	3	4																					
x			6																					
	1	4	0																					
	2	2																						
Year 5	<p>Multiplication of 2 digit by 2 digit numbers using the area model (grid method)</p>	<p>Ensure that the area model reflects the correct proportions. Use of base 10 and counters.</p> <p>23 x 22</p> 	<p>See next page for worked example.</p>																					



44 x 32



x	40	4
30	1,200	120
2	80	8

	<p>Long multiplication – multiplying up to 4 digit numbers by 2 digit numbers.</p>	<p>Concrete and pictorial representations of the grid model (area model) to support the link to long multiplication.</p> <p>Explicit links to place value to be made whilst using long multiplication. E.g. 23×14 3×4 20×4 3×10 10×10</p>	<p>Explicit links to expanded form and area model using brackets. Knowledge of place value.</p> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div>
--	--	--	---

Multiplying decimal numbers (up to 3 decimal places) by integers.

Complete in many contexts – i.e. money and measure.

Use of place value counters.

$$1.212 \times 3$$

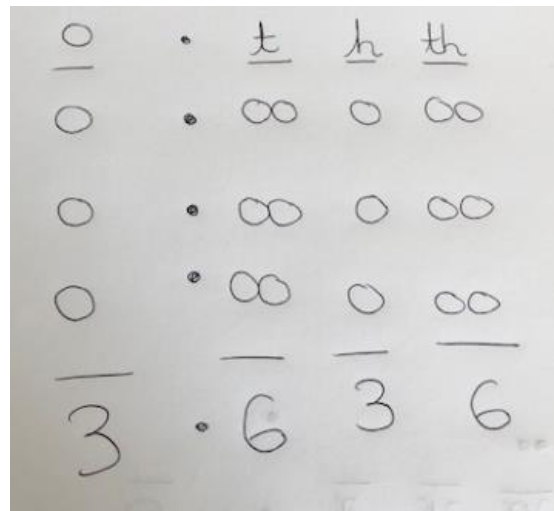
Tens	Ones	Tenths	Hundredths	Thousandths
	1	0.1 0.1	0.01	0.001 0.001
	1	0.1 0.1	0.01	0.001 0.001
	1	0.1 0.1	0.01	0.001 0.001

$$3 \cdot 6 \quad 3 \quad 6$$

Above, we can see 3 equal groups of 1.212.

There are 3 lots of 1.212.

Pupils to complete calculations involving exchanges – see Year 4 for visual representations of exchanges.



Complete as an expanded form first:

$$\begin{array}{r} 3 \cdot 4 \cdot 5 \\ \times \quad \quad 6 \\ \hline 0 \cdot 3 \cdot 0 \\ 2 \cdot 4 \cdot 0 \\ 1 \cdot 8 \cdot 0 \cdot 0 \\ \hline 2 \cdot 0 \cdot 7 \cdot 0 \end{array}$$

Move onto short multiplication method:

$$3.45$$

$$\times 6$$

$$\begin{array}{r} 3.45 \\ \times 6 \\ \hline 20.70 \\ \hline 23 \end{array}$$

Conceptual variation; different ways to ask children to solve 6×23

<table><tr><td>23</td><td>23</td><td>23</td><td>23</td><td>23</td><td>23</td></tr><tr><td colspan="6"></td></tr></table> <p>?</p>	23	23	23	23	23	23							<p>Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?</p> <p>With the counters, prove that $6 \times 23 = 138$</p>	<p>Find the product of 6 and 23</p> <p>$6 \times 23 =$</p> <p><table><tr><td><div></div></td><td>$= 6 \times 23$</td></tr><tr><td><table><tr><td>6</td><td>23</td></tr><tr><td>$\times 23$</td><td>$\times 6$</td></tr><tr><td><hr/></td><td><hr/></td></tr></table></td><td></td></tr></table></p>	<div></div>	$= 6 \times 23$	<table><tr><td>6</td><td>23</td></tr><tr><td>$\times 23$</td><td>$\times 6$</td></tr><tr><td><hr/></td><td><hr/></td></tr></table>	6	23	$\times 23$	$\times 6$	<hr/>	<hr/>		<p>What is the calculation? What is the product?</p> <table><tr><th>100s</th><th>10s</th><th>1s</th></tr><tr><td></td><td><table><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr></table></td><td><table><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr></table></td></tr></table>	100s	10s	1s		<table><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr></table>	●	●	●	●	●	●	●	●	●	●	●	●	<table><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr></table>	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●
23	23	23	23	23	23																																																											
<div></div>	$= 6 \times 23$																																																															
<table><tr><td>6</td><td>23</td></tr><tr><td>$\times 23$</td><td>$\times 6$</td></tr><tr><td><hr/></td><td><hr/></td></tr></table>	6	23	$\times 23$	$\times 6$	<hr/>	<hr/>																																																										
6	23																																																															
$\times 23$	$\times 6$																																																															
<hr/>	<hr/>																																																															
100s	10s	1s																																																														
	<table><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td></tr></table>	●	●	●	●	●	●	●	●	●	●	●	●	<table><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td></tr></table>	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●																													
●	●																																																															
●	●																																																															
●	●																																																															
●	●																																																															
●	●																																																															
●	●																																																															
●	●	●																																																														
●	●	●																																																														
●	●	●																																																														
●	●	●																																																														
●	●	●																																																														
●	●	●																																																														
●	●	●																																																														

Key Vocabulary: multiply, multiplied by, multiplication, times, repeated addition, lots of, groups of, array, double, product of